4-3 Combining Rational Expressions with Addition and Subtraction

Occasionally it will be important to be able to combine two or more rational expressions by addition. Keep in mind two key principles that dictate fraction addition.

Two Guidelines for Addition and Subtraction of Fractions

1. Fractions must have a common denominator.
2. Denominators can only be changed by multiplying the overall fraction by one.

***Exercise* #1:** Combine each of the following fractions by first finding a common denominator. Express your answers in simplest form.

(a) $\dfrac{2x-5}{4x}+\dfrac{4x+2}{6x}$ (b) $\dfrac{4x-1}{5x}+\dfrac{x+5}{10}$ (c) $\dfrac{3}{4x}+\dfrac{1}{2x^2}$

Each of the combinations in *Exercise* #1 should have been reasonably easy because each denominator was monomial in nature. If this is not the case, then it is wise to **factor** the denominators before trying to find a common denominator.

***Exercise* #2:** Combine each of the following fractions by factoring the denominators first. Then find a common denominator and add.

(a) $\dfrac{4}{5y-15}+\dfrac{5}{y^2-9}$ (b) $\dfrac{x-3}{x^2-9x+20}+\dfrac{2}{x^2-6x+8}$

Subtraction of rational expressions is especially challenging because of errors that naturally arise when students forget to distribute the subtraction (or the multiplication by -1). Still, with careful execution, these problems are no different than their addition counterparts.

***Exercise* #3:** Perform each of the following subtraction problems. Express your answers in simplest form.

(a) $\dfrac{3x+7}{x^2-4}-\dfrac{x+3}{x^2-4}$

(b) $\dfrac{x-3}{4x^2-1}-\dfrac{2}{10x+5}$

(c) $\dfrac{x}{x^2-4}-\dfrac{6}{x^2+8x-20}$

(d) $\dfrac{x-2}{x^2+5x+4}-\dfrac{8}{x^2+12x+32}$

***Exercise* #4:** Which of the following is equivalent to $\dfrac{1}{x-1}-\dfrac{1}{x}$?

(1) $\dfrac{x}{x-1}$

(2) $\dfrac{1}{x-x^2}$

(3) $\dfrac{1}{x^2-x}$

(4) $\dfrac{x}{x^2-1}$