4-5 SOLVING FRACTIONAL EQUATIONS

Equations involving fractions or rational expressions arise frequently in mathematics. The key to working with them is to manipulate the equation, typically by multiplying both sides of it by some quantity, that eliminates the fractional nature of the equation. The most common form of this practice is "cross-multiplying."

***Exercise* #1:** Use the technique of cross multiplication to solve each of the following equations.

(a) $\dfrac{4x+5}{2}=\dfrac{x-1}{5}$

(b) $\dfrac{x+1}{2-x}=\dfrac{2}{x-6}$

Since this technique should be familiar to students at this point, we will move onto a less familiar method when more than two fractions are involved. The next exercise will illustrate the process.

***Exercise* #2:** Consider the equation $\dfrac{1}{2}-\dfrac{9}{4x}=\dfrac{3}{4x}$.

(a) What is the least common denominator for all three fractions in this equations?

(b) Multiply both sides of this equation by the LCD to "clear" the equation of the denominators. Now, solve the resulting linear equation.

It is very important to note the similarities and differences between this technique and the one employed to simplify complex fractions. With complex fractions we multiplied by one in creative ways. Here we are multiplying both sides of an equation by a quantity that removes the fractional nature of the equation.

***Exercise* #3:** Which of the following values of x solves: $\dfrac{x-4}{6}+\dfrac{x-2}{10}=\dfrac{31}{15}$?

(1) $x=14$

(3) $x=-8$

(2) $x=6$

(4) $x=11$

These equations can involve quadratic as well as root expressions. The key, though, remains the same – multiplying both sides of the equation by the same quantity.

***Exercise* #4:** Solve each of the following equations for all values of x.

(a) $\frac{1}{10}-\frac{1}{x}=\frac{1}{5x}-\frac{2}{x^2}$

(b) $\frac{1}{2}+\frac{3}{x}-\frac{1}{x^2}=\frac{1}{4x}+\frac{1}{2x^2}$

Because fractional equations often involve denominators containing variables, it is important that we check to see if any solutions to the equation make it undefined. These represent further examples of **extraneous roots**.

***Exercise* #5:** Solve and reject any extraneous roots.

(a) $\frac{x+1}{x+5}+\frac{18}{x^2+8x+15}=\frac{9}{x+3}$

(b) $\frac{4}{x^2+4x-12}+\frac{x-1}{x+6}=\frac{1}{x-2}$