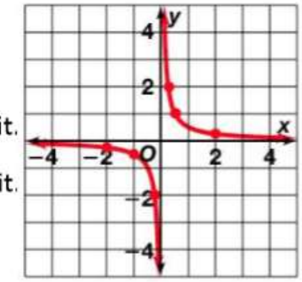
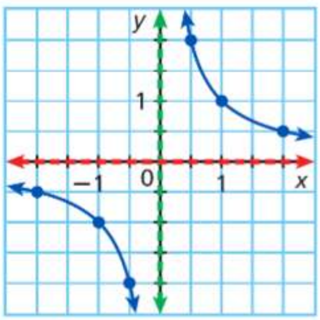
GRAPHING RATIONAL EQUATIONS

$$y = \frac{k}{x}$$



The graph of Rationals has two parts. Each part is called a **branch**.

- As x approaches ___________, the graph gets closer and closer to the x-axis, but never touches it.
- As x approaches ___________, the graph gets closer and closer to the y-axis, but never touches it.



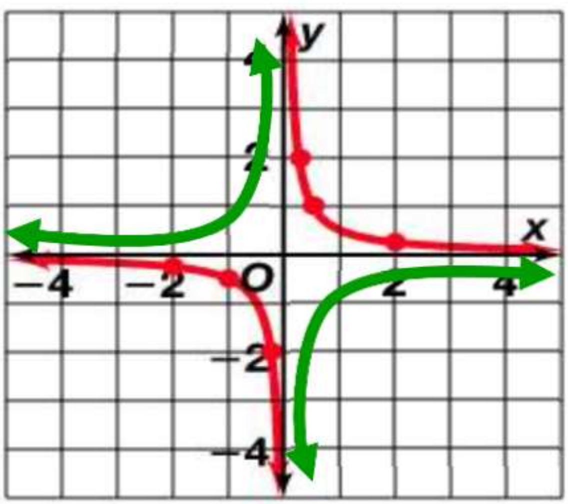
The **asymptotes** of the graph are lines that the graph *approaches*, but never touches.

- The graph has a **vertical** asymptote at ___________.
- The graph has a **horizontal** asymptote at ___________.

Domain: ________________________

Range: ________________________

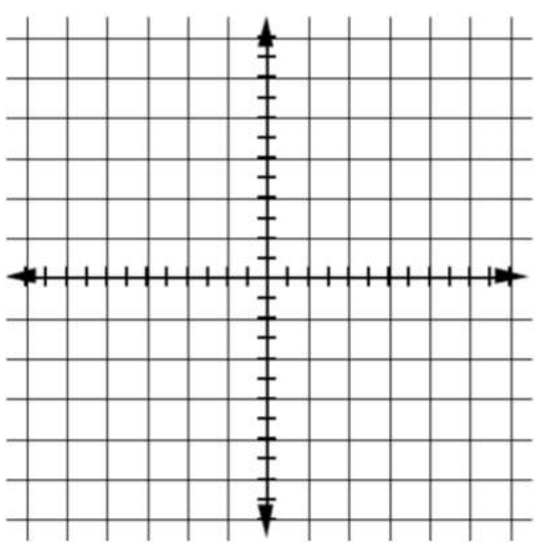
GRAPHING RATIONAL EQUATIONS



When k is **positive**, the branches of   are in **Quadrants** ______ and ______.

When k is **negative**, the branches of   are in **Quadrants** ______ and ______.

Both graphs are symmetric with respect to $y = x$ and $y = -x$.



Ex.1: **Graphing Inverse Variation**

$$y = \frac{6}{x}$$

A **rational function** is any function that can be written as **f(x) =** where **x** ≠ 0.

I. DISCONTINUITIES

Discontinuities are ___________________ in the graph of a function.

The graph of a **polynomial function** is _________________________ (no jumps or breaks).

However, the graph of a **rational function** may be _________________________.

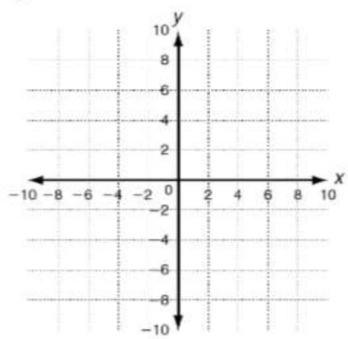
To find these values, **factor** the top and bottom and see if any factors cancel. **There are two cases:**

(1) A factor cancels, which creates a ___________________.

(2) A factor left in the denominator creates a ____________________________________.

Ex. 1: Identifying Holes in a Graph

Identify holes in the graph of $f(x) = \frac{x^2 - 9}{x - 3}$. Then graph.



Ex. 2: Identifying Vertical Asymptotes

Identify the vertical asymptote(s) of

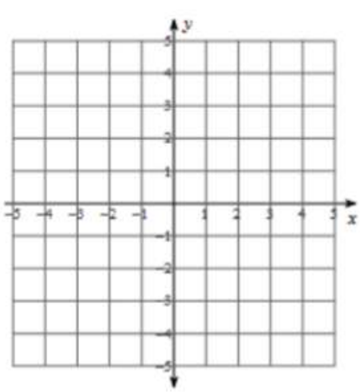
$$f(x) = \frac{x^2+3x-4}{x+3}$$

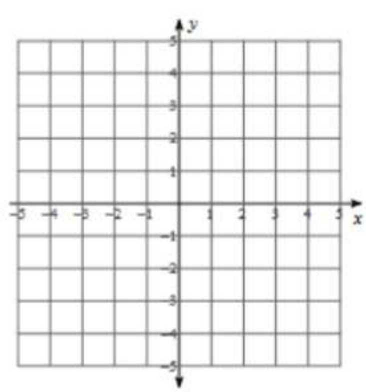
Ex. 3: Finding Vertical Asymptotes and Holes

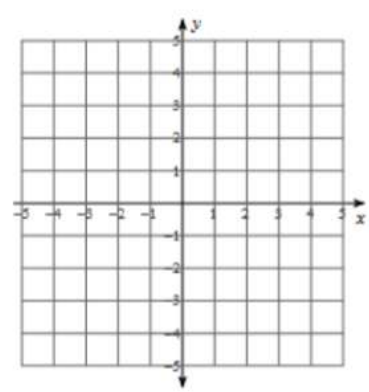
Describe the vertical asymptotes and holes for the graph of each rational function. Then sketch the graph

(Factor the top and bottom. Look for terms that cancel out.)

a. $y = \frac{x + 1}{x^2 + x - 2}$ **b.** $y = \frac{x^2 - x - 2}{x - 2}$ **c.** $y = \frac{x - 2}{x^2 - 3x + 2}$







II. HORIZONTAL ASYMPTOTES

The graph of a function has *at most* ______________ **horizontal asymptote**. To find a horizontal asymptote, compare the degree of the polynomial on top with the degree of the polynomial on the bottom. There are **three Cases**:

(1) If the degree of the denominator (bottom) is **greater than** the degree of the numerator, then _______________ is the horizontal asymptote.

(2) IF the degrees are **equal**, then the graph has a horizontal asymptote at _______________.

Note: *a* is the leading coefficient of the numerator and *b* is the leading coefficient of the denominator.

(3) If the degree of denominator is **less than** the degree of numerator, then there is __________ horizontal asymptote.

B

O

(1) $y = \dfrac{1}{x^2 - 16}$

S

(2) $y = \dfrac{3x^2 - 1}{x^2 + 3}$

T

O

(3) $y = \dfrac{3x^2 - 1}{x + 4}$

N

Ex. 4: **Find any Horizontal Asymptotes:**

a) y = $\dfrac{-4x+3}{2x}\ \dfrac{-4x+3}{2x+1}$

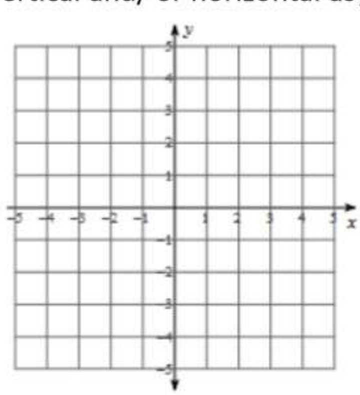
b) y = $\dfrac{5x}{2x^2+x}$

c) y = $\dfrac{4x^2+x-3}{4x+3}$

TRY THESE!!

- Name the values where the function is discontinuous.
- Identify any holes, vertical and/ or horizontal asymptotes. **Then sketch the graph.**

1. y = $\dfrac{3x^2+3x-18}{x-2}$



2. y = $\dfrac{2x^2+6x}{x^3-9x}$