Name: ______________________________ Date: ________________

EXPONENTIAL FUNCTION BASICS
COMMON CORE ALGEBRA II

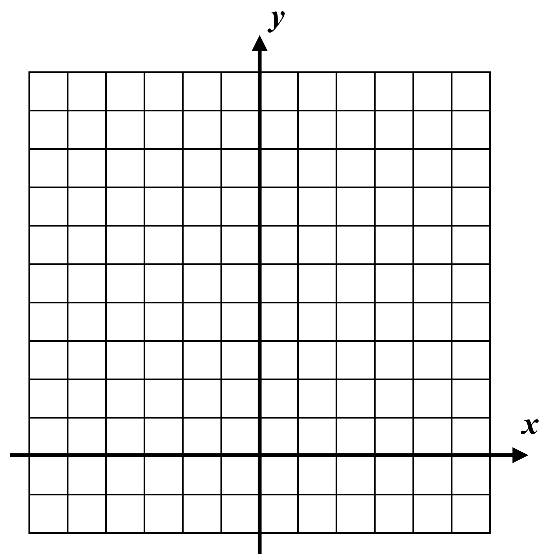
You studied exponential functions extensively in Common Core Algebra I. Today's lesson will review many of the basic components of their graphs and behavior. Exponential functions, those whose exponents are variable, are extremely important in mathematics, science, and engineering.

BASIC EXPONENTIAL FUNCTIONS

$y = b^x$ where $b > 0$ and $b \neq 1$

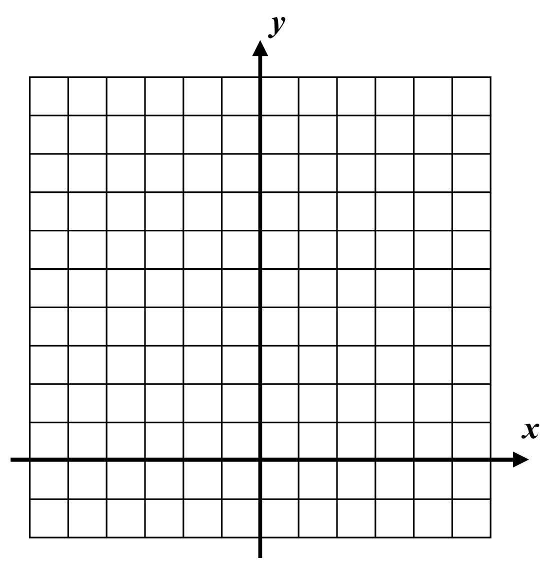
***Exercise* #1:** Consider the function $y = 2^x$. Fill in the table below without using your calculator and then sketch the graph on the grid provided.

x	$y = 2^x$
-3	
-2	
-1	
0	
1	
2	
3	



***Exercise* #2:** Now consider the function $y = \left(\frac{1}{2}\right)^x$. Using your calculator to help you, fill out the table below and sketch the graph on the axes provided.

x	$y = \left(\frac{1}{2}\right)^x$
-3	
-2	
-1	
0	
1	
2	
3	



***Exercise* #3:** Based on the graphs and behavior you saw in *Exercises* #1 and #2, state the domain and range for an exponential function of the form $y = b^x$.

Domain (input set): Range (output set):

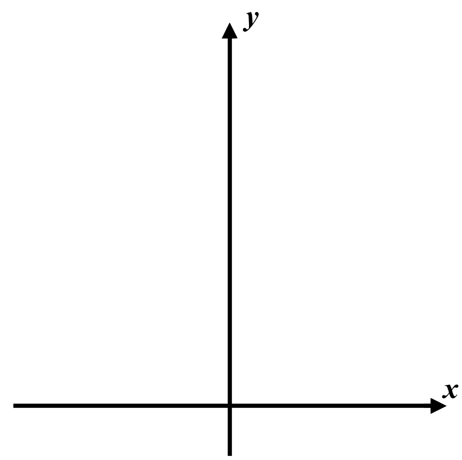
***Exercise* #4:** Are exponential functions one-to-one? How can you tell? What does this tell you about their inverses?

***Exercise* #5:** Now consider the function $y = 7(3)^x$.

(a) Determine the y-intercept of this function algebraically. Justify your answer.

(b) Does the exponential function increase or decrease? Explain your choice.

(c) Create a rough sketch of this function, labeling its y-intercept.

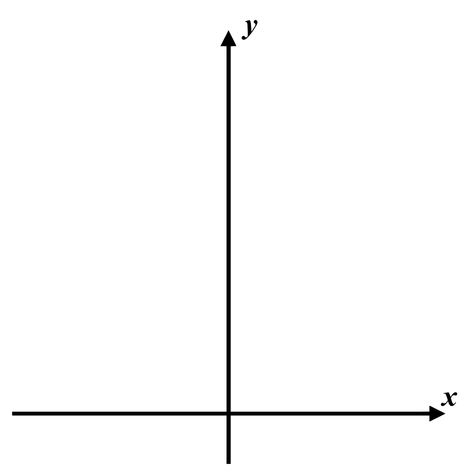


***Exercise* #6:** Consider the function $y = \left(\frac{1}{3}\right)^x + 4$.

(a) How does this function's graph compare to that of $y = \left(\frac{1}{3}\right)^x$? What does adding 4 do to a function's graph?

(b) Determine this graph's y-intercept algebraically. Justify your answer.

(c) Create a rough sketch of this function, labeling its y-intercept.



EXPONENTIAL MODELING WITH PERCENT GROWTH AND DECAY
COMMON CORE ALGEBRA II

Exponential functions are very important in modeling a variety of real world phenomena because certain things either increase or decrease by **fixed percentages** over given units of time. You looked at this in Common Core Algebra I and in this lesson we will review much of what you saw.

***Exercise* #1:** Suppose that you deposit money into a savings account that receives 5% interest per year on the amount of money that is in the account for that year. Assume that you deposit $400 into the account initially.

(a) How much will the savings account increase by over the course of the year?

(b) How much money is in the account at the end of the year?

(c) By what single number could you have multiplied the $400 by in order to calculate your answer in part (b)?

(d) Using your answer from part (c), determine the amount of money in the account after 2 and 10 years. Round all answers to the nearest cent when needed.

(e) Give an equation for the amount in the savings account $S(t)$ as a function of the number of years since the $400 was invested.

(f) Using a table on your calculator determine, to the nearest year, how long it will take for the initial investment of $400 to double. Provide evidence to support your answer.

The thinking process from *Exercise* #1 can be generalized to any situation where a quantity is increased by a fixed percentage over a fixed interval of time. This pattern is summarized below:

INCREASING EXPONENTIAL MODELS

If quantity *Q* is known to increase by a fixed percentage *p*, in decimal form, then *Q* can be modeled by

$$Q(t) = Q_0(1+p)^t$$

where Q_0 represents the amount of *Q* present at $t = 0$ and *t* represents time.

***Exercise* #2:** Which of the following gives the savings *S* in an account if $250 was invested at an interest rate of 3% per year?

(1) $S = 250(4)^t$

(2) $S = 250(1.03)^t$

(3) $S = (1.03)^t + 250$

(4) $S = 250(1.3)^t$

Decreasing exponentials are developed in the same way, but have the percent subtracted, rather than added, to the base of 100%. Just remember, you are ultimately multiplying by the percent of the original that you will have after the time period elapses.

***Exercise* #3:** State the multiplier (base) you would need to multiply by in order to decrease a quantity by the given percent listed.

(a) 10% (b) 2% (c) 25% (d) 0.5%

DECREASING EXPONENTIAL MODELS

If quantity Q is known to decrease by a fixed percentage p, in decimal form, then Q can be modeled by

$$Q(t) = Q_0(1-p)^t$$

where Q_0 represents the amount of Q present at $t = 0$ and t represents time.

***Exercise* #4:** If the population of a town is decreasing by 4% per year and started with 12,500 residents, which of the following is its projected population in 10 years? Show the exponential model you use to solve this problem.

(1) 9,230 (3) 18,503

(2) 76 (4) 8,310

***Exercise* #5:** The stock price of WindpowerInc is increasing at a rate of 4% per week. Its initial value was $20 per share. On the other hand, the stock price in GerbilEnergy is crashing (losing value) at a rate of 11% per week. If its price was $120 per share when Windpower was at $20, after how many weeks will the stock prices be the same? Model both stock prices using exponential functions. Then, find when the stock prices will be equal graphically. Draw a well labeled graph to justify your solution.