COMPOUND INTEREST
COMMON CORE ALGEBRA II

In the worlds of investment and debt, interest is added onto a principal in what is known as **compound interest**. The percent rate is typically given on a yearly basis, but could be applied more than once a year. This is known as the **compounding frequency**. Let's take a look at a typical problem to understand how the compounding frequency changes how interest is applied.

***Exercise* #1:** A person invests \$500 in an account that earns a **nominal yearly interest rate** of 4%.

(a) How much would this investment be worth in 10 years if the **compounding frequency** was once per year? Show the calculation you use.

(b) If, on the other hand, the interest was applied four times per year (known as quarterly compounding), why would it not make sense to multiply by 1.04 each quarter?

(c) If you were told that an investment earned 4% per year, how much would you assume was earned per quarter? Why?

(d) Using your answer from part (c), calculate how much the investment would be worth after 10 years of quarterly compounding? Show your calculation.

So, the pattern is fairly straightforward. For a **shorter compounding period**, we get to **apply the interest more often**, but at a **lower rate**.

***Exercise* #2:** How much would \$1000 invested at a nominal 2% yearly rate, compounded monthly, be worth in 20 years? Show the calculations that lead to your answer.

(1) \$1485.95 (3) \$1033.87

(2) \$1491.33 (4) \$1045.32 ________

This pattern is formalized in a classic formula from economics that we will look at in the next exercise.

***Exercise* #3:** For an investment with the following parameters, write a formula for the amount the investment is worth, A, after t-years.

P = amount initially invested

r = nominal yearly rate

n = number of compounds per year

$A(t) =$

The rate in *Exercise* #1 was referred to as **nominal (in name only)**. It's known as this, because you effectively earn more than this rate if the compounding period is more than once per year. Because of this, bankers refer to the **effective rate**, or the rate you would receive if compounded just once per year. Let's investigate this.

***Exercise* #4:** An investment with a nominal rate of 5% is compounded at different frequencies. Give the **effective** yearly rate, accurate to two decimal places, for each of the following compounding frequencies. Show your calculation.

(a) Quarterly (b) Monthly (c) Daily

Practice: 1. You deposit $10,000 in an account that pays 6% interest. Find the balance after 10 years if the interest is compounded

a) quarterly

b) Monthly

2. $2000 is deposited in an account that pays 8% annual interest, compounded monthly. What is the balance after 5 years?

3. A parent, following the birth of a child, wants to make an initial investment that will grow to $10,000 by the child's 20th birthday. Interest is compounded continuously at 8%. What should that initial investment be?

4. Complete the table: Invest $1 for 1 year at 100% compound interest and compare the result.

Annually	Bi-Annually	Quarterly	Monthly	Weekly	Daily	Hourly	Every Minute	Every Second

Conclusion:

Quick write: Write down any symbols that have defined values. Why are these symbols used instead of numbers?

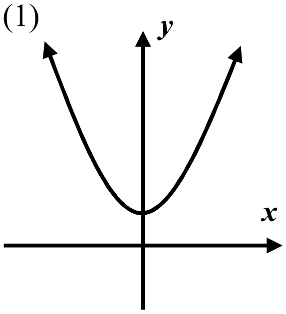
THE NUMBER e AND THE NATURAL LOGARITHM
COMMON CORE ALGEBRA II

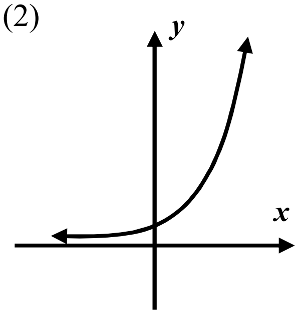
There are many numbers in mathematics that are more important than others because they find so many uses in either mathematics or science. Good examples of important numbers are 0, 1, i, and π. In this lesson you will be introduced to an important number given the letter **e** for its "inventor" Leonhard Euler (1707-1783). This number plays a crucial role in Calculus and more generally in modeling exponential phenomena.

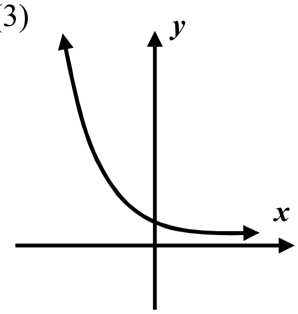
THE NUMBER e

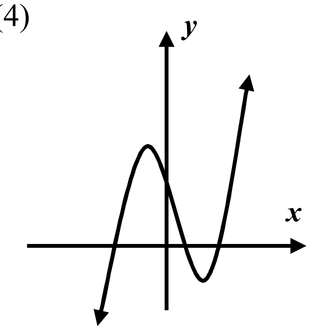
1. Like π, e is irrational.
2. $e \approx 2.72$
3. Used in Exponential Modeling

***Exercise* #1:** Which of the graphs below shows $y = e^x$? Explain your choice. Check on your calculator.

(1)


(2)


(3)


(4)


Explanation:

Very often e is involved in exponential modeling of both increasing and decreasing quantities. The creation of these models is beyond the scope of this course, but we can still work with them.We could compound at smaller and smaller frequency intervals, eventually compounding all moments of time. In our formula from *Exercise* #3, we would be letting n approach infinity. Interestingly enough, this gives rise to **continuous compounding** and the use of the natural base **e** in the famous **continuous compound interest formula.**

CONTINUOUS COMPOUND INTEREST

For an initial principal, P, compounded continuously at a nominal yearly rate of r, the investment would be worth an amount A given by:

$$A(t) = Pe^{rt}$$

***Exercise* #5:** A person invests \$350 in a bank account that promises a nominal rate of 2% continuously compounded.

(a) Write an equation for the amount this investment would be worth after t-years.

(b) How much would the investment be worth after 20 years?

(c) Algebraically determine the time it will take for the investment to reach \$400. Round to the nearest tenth of a year.

(d) What is the effective annual rate for this investment? Round to the nearest hundredth of a percent.

Practice: A student wants to save \$8,000 for college in four years. How much should be put into an account that earns 5.2% annual interest compounded continuously?

5. How long would it take to double your principal at an annual interest rate of 8% compounded continuously?